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Differential Conductance of Type II Superconductors at High Magnetic Fields

Joel Tenenbaum and Sasha Dukan
Department of Physics
Goucher College
1021 Dulaney Valley Road
Baltimore, MD, 21204, USA

Faculty Advisor: Dr. Sasha Dukan

Abstract

The tunneling conductance between the surface of an extreme type-II superconductor and the tip of a Scanning Tunneling Microscope (STM) in high magnetic field with temperature approaching zero is calculated. It is found that when the STM tip is placed at the position of a vortex, the differential conductance σ(V,B) has an algebraic dependence on a bias voltage reflecting the presence of the gapless points in the quasiparticle excitation spectrum of a superconductor in high magnetic fields. The differential conductance as a function of a position of the STM tip and for a fixed value of a bias voltage has a six-fold symmetry of a triangular vortex lattice with the maxima located at the position of the vortices.
Keywords: Superconductivity, High Magnetic Field, Scanning Tunneling Microscopy

1. Motivation

Superconductors are materials that, when cooled below their critical temperatures T_c, will display several remarkable characteristics including zero resistance to passage of an electrical current and a partial or complete expulsion of external magnetic field from its interior, known as the Meissner effect. Type I superconductors typically have critical temperatures below 10K and achieve a full Meissner effect where magnetic field B is repelled from the interior of the sample as long as the field strength is smaller than the critical value $B_{c1}(T)$ (dependent on material), known as a lower critical field. Type II superconductors experience a mixed superconducting/normal state (i.e. partial Meissner effect) for fields B such that $B_{c1}(T) < B < B_{c2}(T)$, where $B_{c2}(T)$ is the upper critical field which also depends on the material under consideration. In this mixed state, type II superconductors channel all magnetic flux into tiny cores called vortices; hence, this state is also called the vortex state. Once the external magnetic field B exceeds $B_{c2}(T)$, the superconducting state is destroyed and the sample reverts to its normal state. Extreme type-II superconductors are defined as systems in which there is a very large difference in magnitude between $B_{c1}(T)$ (~ 10^{-4} Tesla typically) and $B_{c2}(T)$ (larger than 10 Tesla typically), so that the physical behavior of the system is dominated by the vortex state. The properties of such superconductors placed in external magnetic field have been a subject of considerable interest for a long time. The discovery of high-temperature superconductors, exhibiting extreme type-II behavior, has only fueled further and more detailed studies of such systems. This is understandable in light of the possible revolution that high-temperature superconductor applications can bring into the areas of transportation, communication, power grids, etc. The subject of this paper is the vortex state of an extreme type II superconductor placed in high magnetic field and cooled to very low temperatures ($T \rightarrow 0$). In particular, we are interested in how the quasiparticle excitations spectrum in the vortex state influences the behavior of the tunneling conductance between the surface of the sample and the Scanning Tunneling Microscope (STM) tip.

2. The Cooper Pairing on the Landau levels and the Quasiparticle Excitation Spectrum

According to the superconductivity theory developed by Bardeen, Cooper, and Schrieffer (BCS theory), the "mattress" effect of the lattice of positive ions results in an attractive force between pairs of electrons. The coupled electrons are known as Cooper Pairs. When subjected to a fairly high magnetic field (B>>1T), these electrons become quantized into Landau levels with energies $E_n = \hbar\omega_c (n + ½)$ where ω_c=eB/m is the cyclotron frequency, *m* is the effective electronic mass and *n* is the Landau level index. Therefore, any increase in magnetic field strength will result in a proportional increase in separation of Landau levels. As the magnetic field strength approaches its upper limit ($B \rightarrow B_{c2}(T)$), the separation between these levels will become very large as shown in Figure 1.

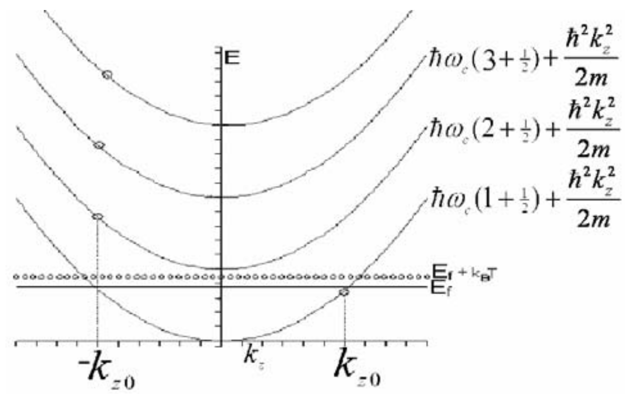

Figure 1. Landau levels of an electron vs. momentum k_z in the direction of magnetic field B.

We can choose an arbitrary electron in the n_0^{th} Landau level with z component of momentum k_{z0} and try to find another electron eligible to pair with it. Because Cooper pairs must have equal and opposite momenta, the latter electron must have z momentum $-k_{z0}$. However, since the only energy levels accessible to electrons will be those below the Fermi energy E_f and those within the reach of the thermal excitations ($E_f + k_BT$), electrons with the necessary momenta are not often found in the same Landau level with index n_0, especially as the magnetic field increases (Landau levels spread further apart) or temperature decreases (thermal excitations are less), as shown in Figure 1. Consequently, it is an excellent approximation to ignore the contributions of these very few electron pairs altogether in the limit as $B \rightarrow B_{c2}(T)$ and $T \rightarrow 0$.

The superconducting order parameter forms a perfect triangular lattice, known as the Abrikosov vortex lattice. In the Landau gauge, where the external vector potential is $\boldsymbol{A}=B(-y,0,0)$ and external magnetic field is $B = \nabla \times A$, the order parameter is given by[1]

$$\Delta(\mathbf{r}) = \Delta \sum_{m=-\infty}^{\infty} exp\left(i\frac{1}{2}\pi m^2 + i2\pi m\frac{x}{a} - \left(\frac{y}{l} + \frac{\pi ml}{a}\right)^2 \right) \tag{1}$$

where Δ is the amplitude of the order parameter, $l = \sqrt{\hbar / Be}$ is the magnetic length, and *a* is the unit cell parameter of the triangular Abrikosov lattice shown in Figure 2. The quasiparticle energy spectrum of a superconductor in the vortex state, where only electrons belonging to the same Landau level were included, as described above, was found to have the form[2]

$$E_{k_z,\mathbf{k},n} = \sqrt{\left[\frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c\left(n+\tfrac{1}{2}\right) - E_f\right]^2 + \left|\Delta_{nn}\left(k_x,k_y\right)\right|^2} \tag{2}$$

where

$$\Delta_{nn}(k_x,k_y)=\frac{1}{\sqrt{2}}\frac{(-1)^n}{2^{2n}n!}\sum_j \exp\left(i\pi\frac{1}{2}j^2+2ijk_y\tfrac{\sqrt{3}}{2}a-\left(k_x+\frac{\pi j}{a}\right)^2 l^2\right)H_{2n}\left[\sqrt{2}\left(k_x+\frac{\pi j}{a}\right)l\right] \qquad (3)$$

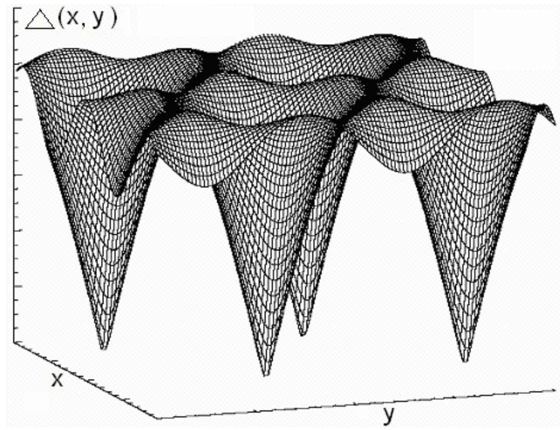

Figure 2. Abrikosov triangular vortex lattice

The function $H_{2n}(x)$ in equation (3) is the Hermite polynomial of order *2n* and $\mathbf{k}=(k_x,k_y)$ is the momentum in the plane perpendicular to the direction of the magnetic field. For a quasiparticle at the Fermi surface,

$$\varepsilon_n(k_z)=\hbar^2k_z^2/2m+\hbar\omega_c\left(n+\tfrac{1}{2}\right)-E_f=0\,, \qquad (4)$$

and the quasiparticle energies reduce to the values of the gaps $\Delta_{nn}(q_x,q_y)$ in equation (3). These gaps go to zero at certain highly symmetrical points on the Fermi surface and have strong linear dispersion around these points[2-4]. This result is in contrast to the familiar result[8] in zero magnetic field where quasiparticle excitations at the Fermi surface are gapped with uniform gap Δ. Therefore, in zero magnetic field, in order to create a current (electrical, thermal, etc…) in the vortex state, the quasiparticles need to gain at least Δ amount of energy to reach the conduction band; however, in high magnetic field, there will be an abundance of quasiparticle excitations that will be able to carry current even at very low excitation energies. This results in qualitatively different behavior of the electrical conductivity[4], thermal conductivity[5], tunneling current, and many other thermodynamic properties[6] of the extreme type II superconductor at very low temperatures.

3. The Differential Conductance of Type II Superconductors in Vortex State

One of the most powerful probes of the quasiparticle energy spectrum in the mixed state is the measurement of tunneling current between the Scanning Tunneling Microscope (STM) tip and the superconductor's surface. The tunneling current is typically characterized by the differential conductance $\sigma(\mathbf{r},V,T)=dI/dV$ as a function of the bias voltage between the STM tip and the surface[7]. Varying the bias voltage allows us to observe the energy levels of the tunneling quasiparticles. In a zero magnetic field the quasiparticle excitations are gapped with large Δ and there will be no available energy states for the quasiparticles to tunnel into until the bias voltage V is such that $eV > \Delta$. On the other hand, in high magnetic fields we expect a non-zero tunneling current even for small bias voltages due to the presence of low-energy excitations at the Fermi surface around gapless points in the excitation spectrum given by equation (2).

The differential conductance in the vortex state of the superconductor in a magnetic field is given by[2]

$$\frac{\sigma(\mathbf{r},V,T)}{\sigma_0}=-\frac{2\pi l^2}{N_1(0)}\sum_{k_z>0}\sum_{\mathbf{k}}\sum_{n=0}^{n_c}\left[\left|u^n_{\mathbf{k}k_z}(r)\right|^2 n'_f(E^n_{\mathbf{k}k_z}-eV)+\left|v^n_{\mathbf{k}k_z}(r)\right|^2 n'_f(E^n_{\mathbf{k}k_z}+eV)\right] \qquad (5)$$

where σ_0 is the differential conductance of the normal metal and $N_1(0)$ is the one-dimensional density of states of free electrons at the Fermi surface moving along the direction of magnetic field. The summation over Landau level index n in equation (5) is over all occupied levels where n_c is determined as the ratio $E_f / \hbar\omega_c$ rounded to the nearest integer. The derivative of the Fermi-Dirac distribution, $n`_f(E)$ in equation (5), becomes the Dirac δ-function when T→0. The Cooper pair amplitudes $u^n_{\mathbf{k}k_z}(\mathbf{r})$ and $v^n_{\mathbf{k}k_z}(\mathbf{r})$ are given by[8]

$$\left|u^n_{\mathbf{k}k_z}(\mathbf{r})\right|^2 = \frac{1}{2}\left(1+\frac{\varepsilon_n(k_z)}{E^n_{\mathbf{k}k_z}}\right)\left|\phi_{\mathbf{k}k_z n}(\mathbf{r})\right|^2$$
$$\left|v^n_{\mathbf{k}k_z}(\mathbf{r})\right|^2 = \frac{1}{2}\left(1-\frac{\varepsilon_n(k_z)}{E^n_{\mathbf{k}k_z}}\right)\left|\phi_{\mathbf{k}k_z n}(\mathbf{r})\right|^2 \qquad (6)$$

where the wavefunctions $\phi_{\mathbf{k}k_z n}(\mathbf{r})$ of the electrons in the magnetic field in the presence of the vortex lattice can be determined as[2]

$$\phi_{\mathbf{k}k_z n}(\mathbf{r}) = \frac{1}{\sqrt{2^n n!\sqrt{\pi} l}}\sqrt{\frac{\frac{\sqrt{3}}{2}a}{L_x L_y L_z}}\exp(ik_z z)\sum_m \exp\left(i\frac{\pi}{4}m^2 - imk_y \tfrac{\sqrt{3}}{2}a\right)$$
$$\times\exp\left\{i\left(k_x+\tfrac{\pi m}{a}\right)x - \tfrac{1}{2}\left[\frac{y}{l}+\left(k_x+\tfrac{\pi m}{a}\right)l\right]^2\right\}H_n\left[\tfrac{y}{l}+\left(k_x+\tfrac{\pi m}{a}\right)l\right] \qquad (7)$$

where L_x, L_y, L_z are the macroscopic dimensions of the system. Taking into the account equations (6) and (7) and performing the continuous limit in the summation over the momenta, equation (4) reduces to

$$\frac{\sigma(\mathbf{r},V,T)}{\sigma_0} = \frac{1}{16}\sum_{n=0}^{n_c}\sqrt{\left(\frac{-\hbar\omega_c n}{E_f}-\frac{\hbar\omega_c n}{2E_f}+1\right)}\int_{-1}^{1}\int_{-1}^{1} dr_{k1}dr_{k2}\left|\phi_{\mathbf{k}k_z n}(\mathbf{r})\right|^2\frac{|eV|}{\sqrt{(eV)^2-\left|\Delta_{nn}(r_{k1},r_{k2})\right|^2}} \qquad (8)$$

where $\mathbf{r}$=(x,y) represents the position of the STM tip.

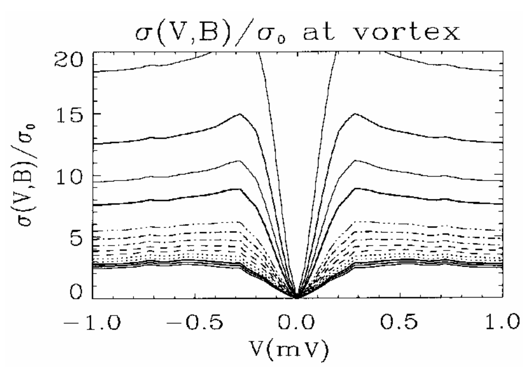

Figure 3. Differential Conductance vs. Voltage V for $YNi_2B_2C_2$ superconductor with the STM tip at position of a vortex site. The differential conductance decreases as the magnetic field is increases from B=2 Tesla to B=8 Tesla in increments of .5 T.

We compute the differential conductance using equation (8) for a borocarbide superconductor $YNi_2B_2C_2$ known for its extreme type II behavior. We take the experimentally determined amplitude of the order parameter $\Delta=2.3\ meV$, effective mass $m=.35m_e$, Fermi velocity $v_f=2.5 \times 10^7\ cm/s$ from Reference 9 and calculate the Fermi Energy to be $E_f=0.062\ eV$ as well as $n_c\sim 25$ at $B=1$ *Tesla* and $n_c\sim 200$ at $B_{c2}=8$ *Tesla*. Figure 3 shows the differential conductance in equation (6) as a function of the bias voltage V when the STM tip is at the position of the vortex for $YNi_2B_2C_2$ superconductor placed in magnetic fields varying from $B=2$ *Tesla* to $B=8$ *Tesla* in increments of *0.5 Tesla*. As the result of the linear dispersion around gapless points in the excitation spectrum, the differential conductance has a linear dependence on *V* at low values. Furthermore, the differential conductance increases as the magnetic field decreases, which reflects the increase in the number of occupied Landau levels that participate in the Cooper pairing. In Figure 4, the differential conductance from equation (8) is plotted as a function of the position of the STM tip with the bias voltage fixed at $V=0.46\ mV$. It can be concluded by inspection that this figure has a six-fold symmetry of the triangular vortex lattice shown in Figure 1. The differential conductance has maxima at positions of the vortices and decreases as the STM tip moves away from the vortex. It reaches its minimum value at the positions where the order parameter $\Delta(\mathbf{r})$ has its maxima (these points form the hexagonal lattice).

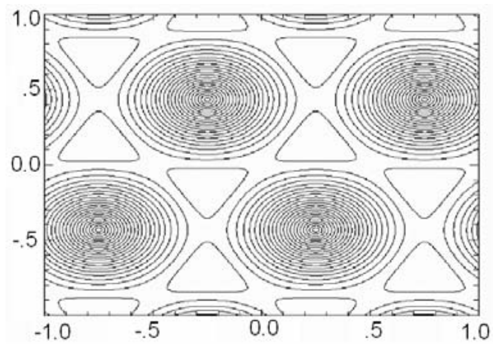

Figure 4. The Differential conductance vs. the position of the STM tip in the $YNi_2B_2C_2$ superconductor where the value of the bias voltage is $V/\Delta=0.2$ and the strength of magnetic field is B=7 Tesla.

4. Conclusions

In this work we predict that the differential conductance σ(V,B) of an extreme type II superconductor such as borocarbide superconductor YNi_2B_2C will have a linear dependence on a bias voltage *V* for small values of *V* when a position of the STM tip is fixed. This is a consequence of the presence of the gapless points in the quasiparticle excitation spectrum of a superconductor in high magnetic fields and at low temperatures. This result is in stark contrast to the σ(V,B) behavior at a zero magnetic field where there will be no tunneling current for small *V*. We also find that the differential conductance as a function of the position of the STM tip, for a fixed value of bias voltage and magnetic field, exhibits a six-fold symmetry of a triangular vortex lattice with the maxima located at the position of the vortices. This unusual behavior of extreme type II superconductors in high magnetic field should be readily detectable in a suitably designed STM experiment at very low temperatures.

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6. References

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